NAME: ____
(Print) Last Name, First Name

STUDENT NUMBER: ______________________________

The Irving K. Barber School of Arts and Sciences

Physics 232— Winter 2014/2015 – Term 2
FINAL EXAMINATION

Instructor: Jake Bobowski

Monday, April 21, 2015 Time: 18:00 - 21:00
Location: EME 2111

This Examination was prepared by Jake Bobowski
Not including this coversheet, the exam consists of 18 numbered pages.

- **Attempt any 6 of the 9 problems.**
- **Put a cross through three of numbers that are listed just below to indicate the problems that you do not wish to be graded**

1 2 3 4 5 6 7 8 9

If necessary, you may use the backs of pages for calculations.
You must clearly show your work to receive full credit.
Writing down only the correct final answer will not earn full credit.

1	2	3	4	5	6	7	8	9	total
10	10	10	10	10	10	10	10	10	60

Free Response: Write out complete answers to the following questions. Show your work.

(10pts) **1.** (a) Bits are sent over a communications channel in packets of 12. If the probability of a bit being corrupted over this channel is 0.1 and such errors are independent, what is the probability that no more than 2 bits in a packet are corrupted? (**5 marks**)

(b) If 6 packets are sent over the channel, what is the probability that at least one packet will contain 3 or more corrupted bits? (**5 marks**)

10 pts

0 pts

(10pts) **2.** The Poisson distribution is given by:

$$P_{\mathrm{P}}(x;\mu) = \frac{\mu^x}{x!}e^{-\mu}$$

(a) Determine $\langle x \rangle$, the mean value of x for this distribution. *Hint*: After writing down the appropriate sum and making some initial manipulations, make use of the substitution $y = x - 1$. (**5 marks**)

(b) Determine $\langle x^2 \rangle$, the mean value of x^2 for this distribution. *Hint*: After writing down the appropriate sum and making some initial manipulations, make use of the substitution $y = x - 1$ and the result of part (a). (**5 marks**)

For both parts (a) and (b), you must show all steps and all of your work. Writing down only the correct final answers will result in 1 out of a possible 5 marks for each part.

10 pts

0 pts

(10pts) **3.** The goal of this problem is to describe how to analyze a set of impedance versus frequency data to extract the resistance R and capacitance C of an RC-series circuit.

The impedance of a series RC circuit is given by:

$$|Z| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

where $\omega = 2\pi f$ and f is frequency.

A researcher measures the current in the circuit that results from applying ac voltages at a number of different frequencies. From these data the frequency dependence of the impedance $|Z| = |v|/|i|$ is determined from 1 to 10 kHz as given in the table on the following page. Assume that the uncertainty in f is negligible.

f (kHz)	$\|Z\|$ (kΩ)	$\sigma_{\|Z\|}$ (kΩ)	x	y	σ_y
1.00	1800	100			
2.00	870	50			
5.00	580	40			
7.50	510	40			
10.00	520	30			

(a) Linearize the equation above such that the resistance R and capacitance C can be extracted from the slope and y-intercept of a straight line. Give the equation of the straight line and describe the plot (y vs x) that you would generate. What does y represent and what does x represent? How are R and C related to the y-intercept a and slope b of a linear fit to the data? (**6 marks**)

(b) Complete the three right-hand columns in the table above. That is, calculate the x, y, and σ_y values that you would plot. What are the units of x, y, and σ_y? (**4 marks**)

10 pts

0 pts

(10pts) **4.** The function $f(x)$ is defined to be periodic with a period of 2π. On the interval $-\pi < x < \pi$ the function is defined as $f(x) = \pi^2 - x^2$.

(a) Sketch several periods of $f(x)$. Be sure to include scales for both the $x-$ and $y-$axes of your plot. (**2 marks**)

(b) Find the Fourier series for this function. Simplify your answers as much as possible. Write out the first five non-zero terms of the Fourier series. (**8 marks**)

10 pts

0 pts

(10^{pts}) **5.** If n measurements of a quantity x are made each with its own uncertainty σ (i.e. $x_1 \pm \sigma_1$, $x_2 \pm \sigma_2$, $\ldots$, $x_n \pm \sigma_n$), then the appropriate weighted mean of the measurements is given by:

$$\mu = \frac{\displaystyle\sum_{i=1}^{n} \frac{x_i}{\sigma_i^2}}{\displaystyle\sum_{i=1}^{n} \frac{1}{\sigma_i^2}}$$

(a) Find a general expression for the error in the weighted mean σ_μ. You must show all of your work. Simply writing down the correct final answer will earn only 1 out of a possible 5 marks. (**5 marks**)

(b) Three groups of particle physicists measure the mass of a certain elementary particle with the following results (in units of MeV/c^2): 1967.0 ± 1.0, 1969.0 ± 1.4, and 1972.1 ± 2.5. Find the weighted mean of these measurements and its uncertainty. (**5 marks**)

10 pts

0 pts

(10pts) **6.** In his famous experiment with electrons, J.J. Thompson measured the "charge-to-mass ratio" $r \equiv e/m$, where e is the electron's charge and m its mass. This experiment is done by accelerating electrons through a voltage V and then deflecting their direction in a magnetic field. The charge-to-mass ratio is given by:

$$r = \frac{125}{32\mu_0^2 N^2} \frac{D^2 V}{d^2 I^2}$$

The magnetic field is generated using a coil consisting of N loops where each loop has diameter D and carries current I. When it's deflected, the electron follows a curved path of radius d. If the experimentally measured quantities are:

$$D = 661 \pm 2 \text{ mm}$$
$$V = 45.0 \pm 0.2 \text{ V}$$
$$d = 91.4 \pm 0.5 \text{ mm}$$
$$I = 2.48 \pm 0.04 \text{ A}$$

what is the experimentally determined value of r and its uncertainty? Assume that $N = 72$ and $\mu_0 = 4\pi \times 10^{-7}$ N/A^2 are both known exactly. (**8 marks**)

(b) The known values the electron charge and mass are $e = 1.60 \times 10^{-19}$ C and $m = 9.11 \times 10^{-31}$ kg. Does the experimentally determined value of r agree with the expected value? (**2 marks**)

10 pts

0 pts

(10$^{\text{pts}}$) **7.** In a certain dice game a player rolls a pair of dice. If the player rolls doubles, then he wins \$10, if he doesn't roll doubles and the pair of dice sum to an even number he must pay \$3, if he doesn't roll doubles and the pair of dice sum to an odd number he must pay \$1.50.

So, for example, if the player rolls a pair of 4's he wins \$10. If he rolls a 2 and a 4, he must pay \$3 because it's not a double and $2+4$ is even. If he rolls a 2 and a 5, he must pay \$1.50 because it's not a double and $2+5$ is odd.

(a) If the someone plays many many rounds of this game, what is the average amount of money won (or lost) per turn? (**4 marks**)

(b) What is the standard deviation in the amount paid out per turn? (**3 marks**)

(c) If someone plays the game exactly $N = 100$ times, what is the net amount of money (from all 100 rounds) that the player should expect to win (or lose)? What would be the uncertainty in that net amount? (**3 marks**)

10 pts

0 pts

(10pts) **8.** The goal of this problem is to estimate the value of a definite double integral of a function of two variables:

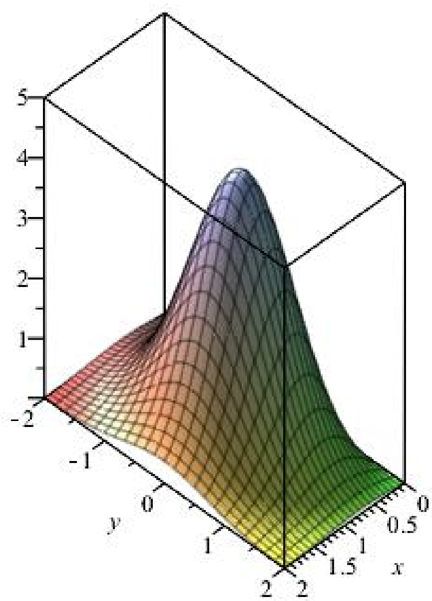
$$\int_{x_\mathrm{i}}^{x_\mathrm{f}} \int_{y_\mathrm{i}}^{y_\mathrm{f}} f(x,y)\,dy\,dx$$

using Monte Carlo methods.

For example, suppose that we want to evaluate the following integral of $f(x,y) = 10x\,e^{-x^2-y^2}$:

$$I = \int_0^2 \int_{-2}^2 10x\,e^{-x^2-y^2}\,dy\,dx$$

where the x- and y-integrals span $0 \le x \le 2$ and $-2 \le y \le 2$ respectively. A plot of $f(x,y)$ over these intervals is shown in the figure below.



(a) Describe in detail an implementation of the Monte Carlo f-average $\left(\bar{f}\right)$ method that could be used to estimate the numerical value of the double integral of $f(x,y)$. (**6 marks**)

(b) Suppose that an f-average Monte Carlo method was implemented and it was found that $\bar{f} = 1.08 \pm 0.11$ over the plane spanned by $0 \le x \le 2$ and $-2 \le y \le 2$.

What would be the resulting estimate for the value of $I \pm \sigma_I$? (**4 marks**)

10 pts

0 pts

(10pts) **9.** Suppose that we have a linear function $y = a + bx$ where a is the y-intercept and b is the slope. If we have a set of measurements $(x_i, y_i \pm \sigma_i)$ where $i = 1, 2, 3 \ldots, N$, then the best-fit values of a and b are determined from a minimization of χ^2:

$$\frac{\partial \chi^2}{\partial a} = 0$$
$$\frac{\partial \chi^2}{\partial b} = 0$$

(a) Write down an expression for χ^2 in terms of a, b, x_i, y_i, and σ_i. (**2 marks**)

(b) What is the expected value of χ^2 for a given set of data and best-fit parameters? Assume that $N \gg 2$. What does it mean if χ^2 turns out to be much larger than expected? What does it mean if χ^2 turns out to be much smaller than expected? (**3 marks**)

(c) Describe in detail the "method of maximum likelihood" used to determine the best-fit values of a and b from a set of measurements. You don't need to formally derive expressions for the best-fit values of a and b. Just start by writing down the probability of obtaining the data set $(x_1, y_1 \pm \sigma_1)$, $(x_2, y_2 \pm \sigma_2), \ldots,$ $(x_N, y_N \pm \sigma_N)$ for a given slope and y-intercept. Then argue that minimizing χ^2 as discussed above leads to the best values of a and b for the given dataset. (**5 marks**)

10 pts

0 pts